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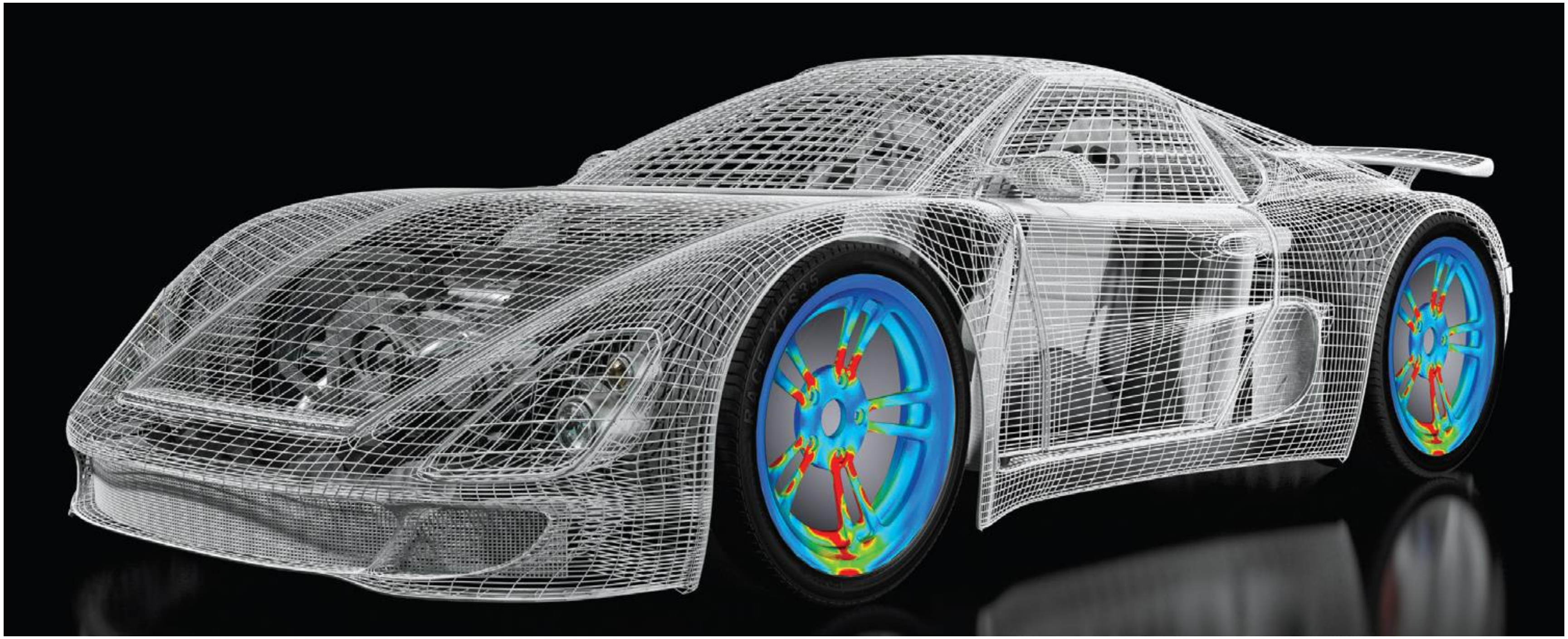
# Mechanical and Aeronautical Numerical Analysis



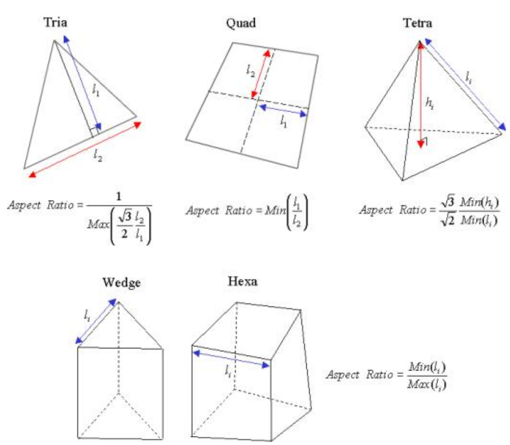
Braverman Arik

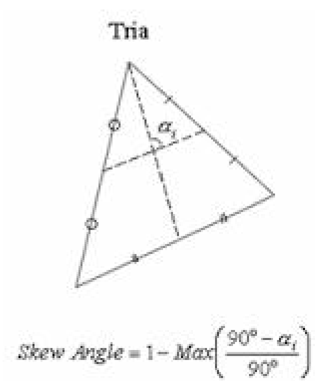
# Agenda

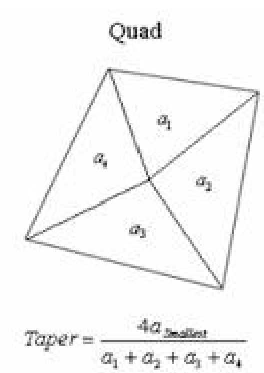
- What is the Mesh Criteria?
- Mesh Free Method
- Mesh Free VS Mesh
- Conclusion

# Mesh Quality

- Aspect Ratio - The aspect ratio refers to the ratio of the longest edge to the shortest edge in the element. The aspect ratio significantly affects the analysis results, and a very small aspect ratio may lead to abnormal analysis results
- Skew Angle - The skew angle refers to an angle deviating from the rectangular shape (90 degrees). Since two sides of a rectangle forms an angle of 90 degrees, the skew angle is 0, which is the most ideal value. The farther away from the shape of a rectangle, the higher the skew angle becomes (to a number greater than 0).

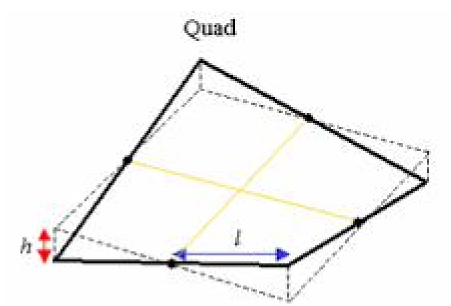


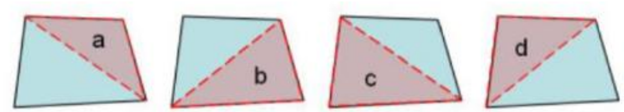



# Mesh Quality

- Warpage - The warpage refers to the state of deviation from a plane.  If all the nodes of a 2D rectangular element are located on the same plane, the warpage is 0, which is the most ideal value.  The farther away from the state of a plane, the greater the warpage angle becomes (to a number greater than 0).
- Taper - The taper refers to the state of deviation calculated geometrically.  Taper is not applicable to triangular elements.  A rectangle retains the taper of 1, which is the most ideal value.  The farther away from the shape of a rectangle (to the shape of a triangle), the smaller the taper becomes (to a number smaller than 1).



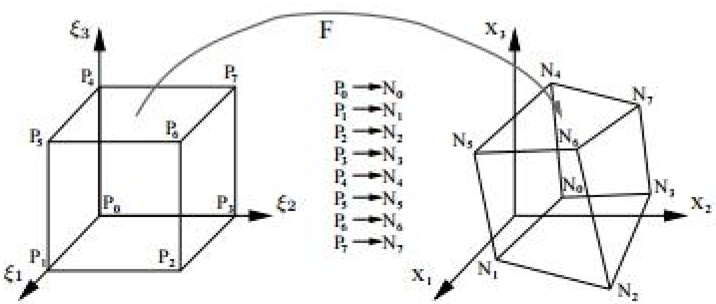

$$Warpage = 1 - \frac{h}{Min(l)}$$

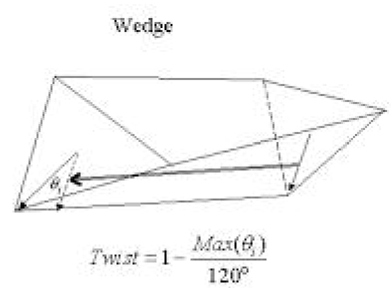


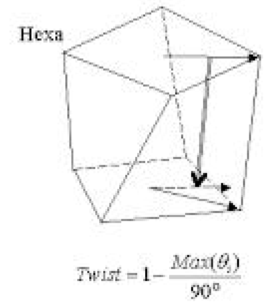

$$taper = 1 - \left( \frac{A_{tri}}{0.5 \times A_{quad}} \right)_{min.}$$

# Mesh Quality

- Jacobian Ratio - The Jacobian ratio refers to the ratio of the smallest Jacobian determinant to the greatest Jacobian determinant. The higher the Jacobian ratio, the better It is
- Twist Angle - The twist refers to the state of twist between 2 opposing faces of a solid element
- Element Length - Check the lengths of edges of an element. The minimum and maximum values can be specified



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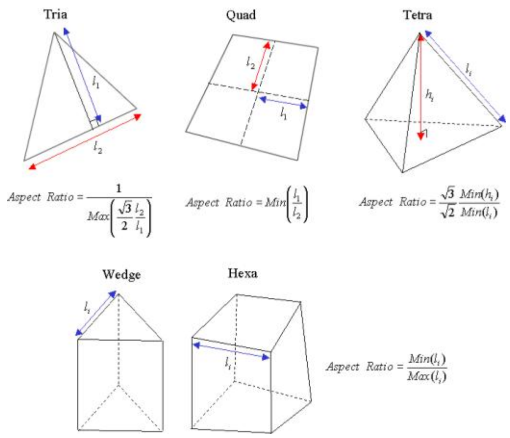


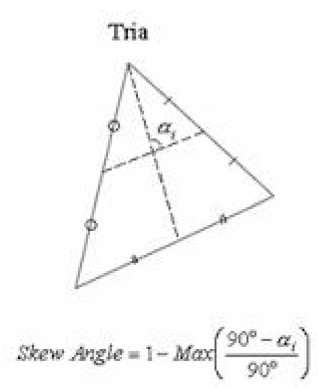


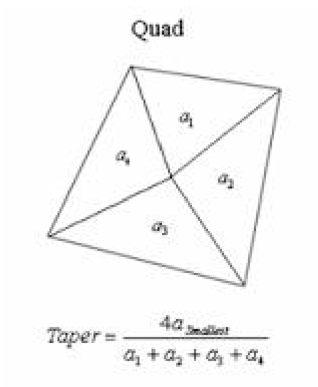

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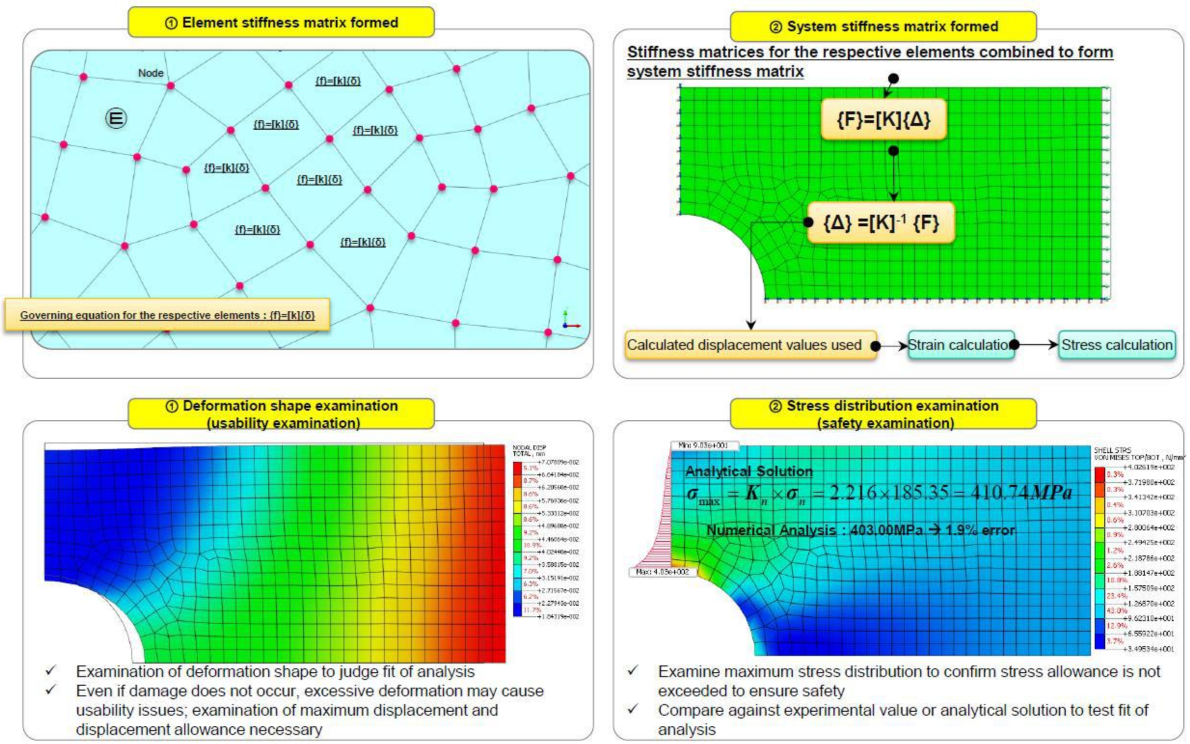
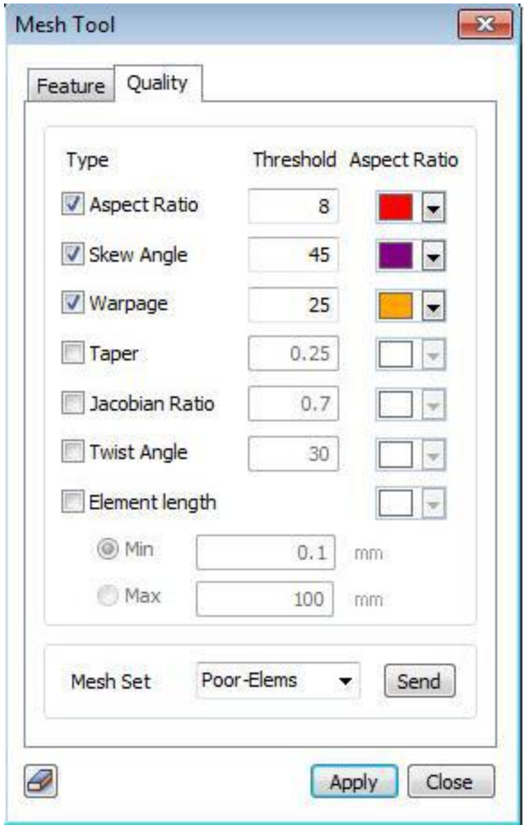
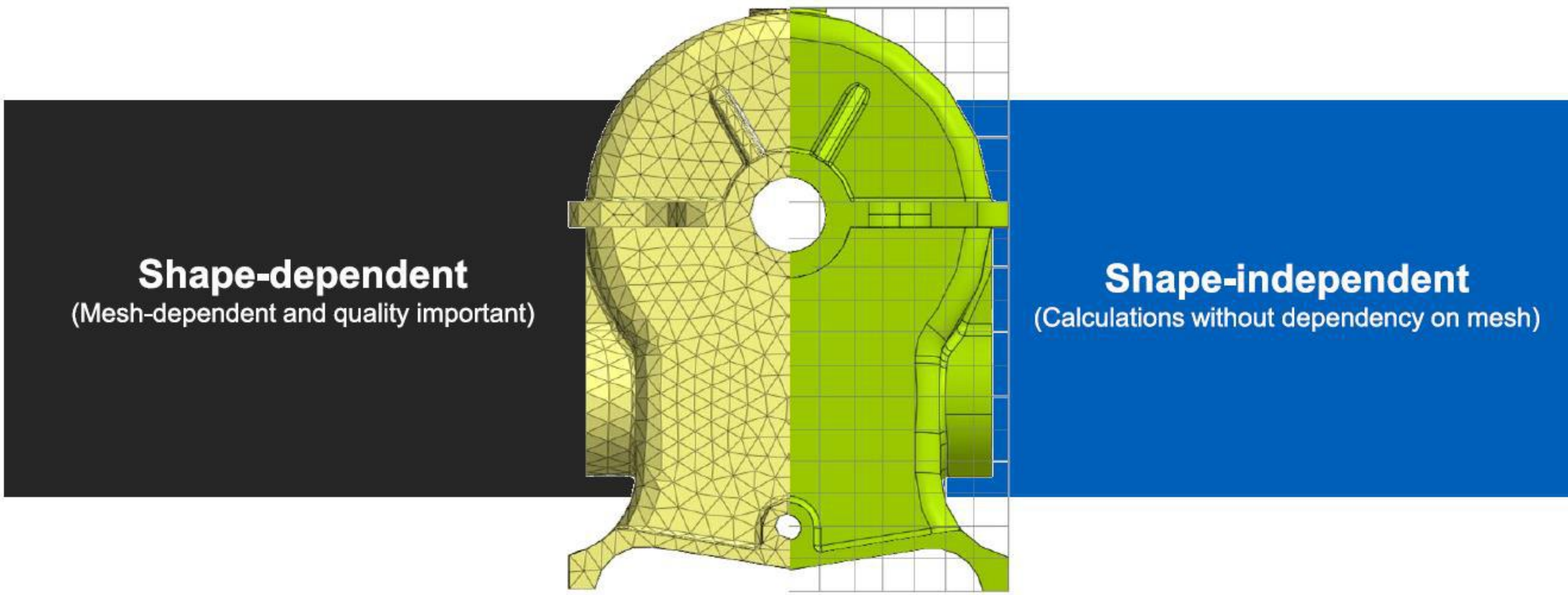
# Mesh Quality



# Free Mesh


Shape-dependent
(Mesh-dependent and quality important)
Shape-independent
(Calculations without dependency on mesh)

# Free Mesh Steps

## Analytical process (MeshFree)

Shorter overall analysis time compared to FEM method and high analysis success rate - Various post-processing functions for user convenience

**STEP1**

**Direct CAD Interface (definition of material)**

**STEP2**

**Definition of load/boundary conditions**

**STEP3**

**Analysis and results using state-of-the-art technology**

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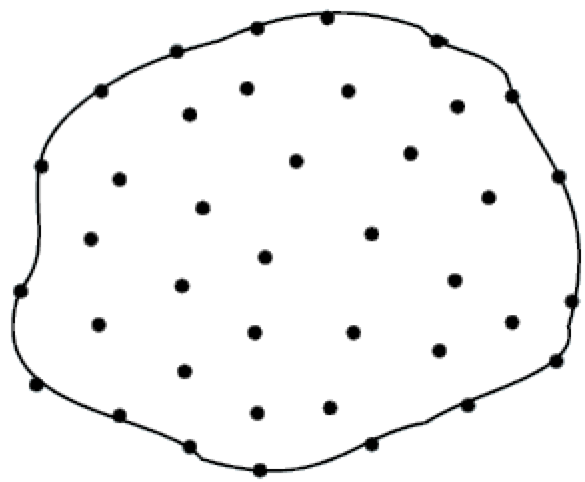
# What is Mesh Free

Mesh Free Methods use a set of nodes scattered within the problem domain as well as sets of nodes scattered on the boundaries of the domain to **represent** (not discretize) the problem domain and its boundaries.

No mesh implies no information on the relationship between the nodes is required.



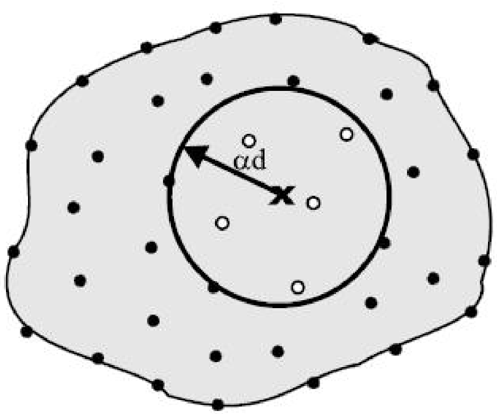
# Mesh Free Support Domain

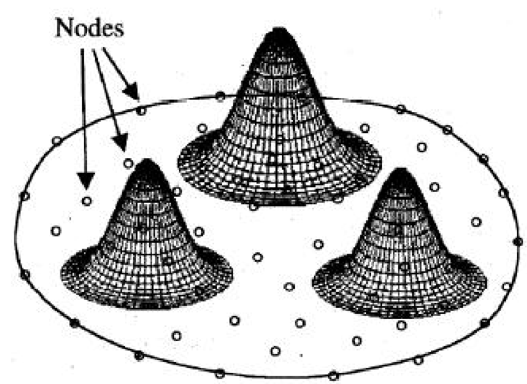
- *Definition*
  The support domain for a point $\boldsymbol{x}$ is a sphere of a certain radius that relates to the nodal spacing near the point $\boldsymbol{x}$.
- *Reason*
  Determines the number of nodes to be used to approximate the function value at $\boldsymbol{x}$.
- *Restriction*
  The nodal density does not vary drastically in the problem domain.

# Mesh Free Field Interpolation

The field variable $u$ at any point $\boldsymbol{x}$ within the problem domain is interpolated using the values of this field at all the nodes within the support domain of $\boldsymbol{x}$. Mathematically,

$$u(\boldsymbol{x}) = \sum_{j=1}^{n} \phi_i(\boldsymbol{x}) u_i$$



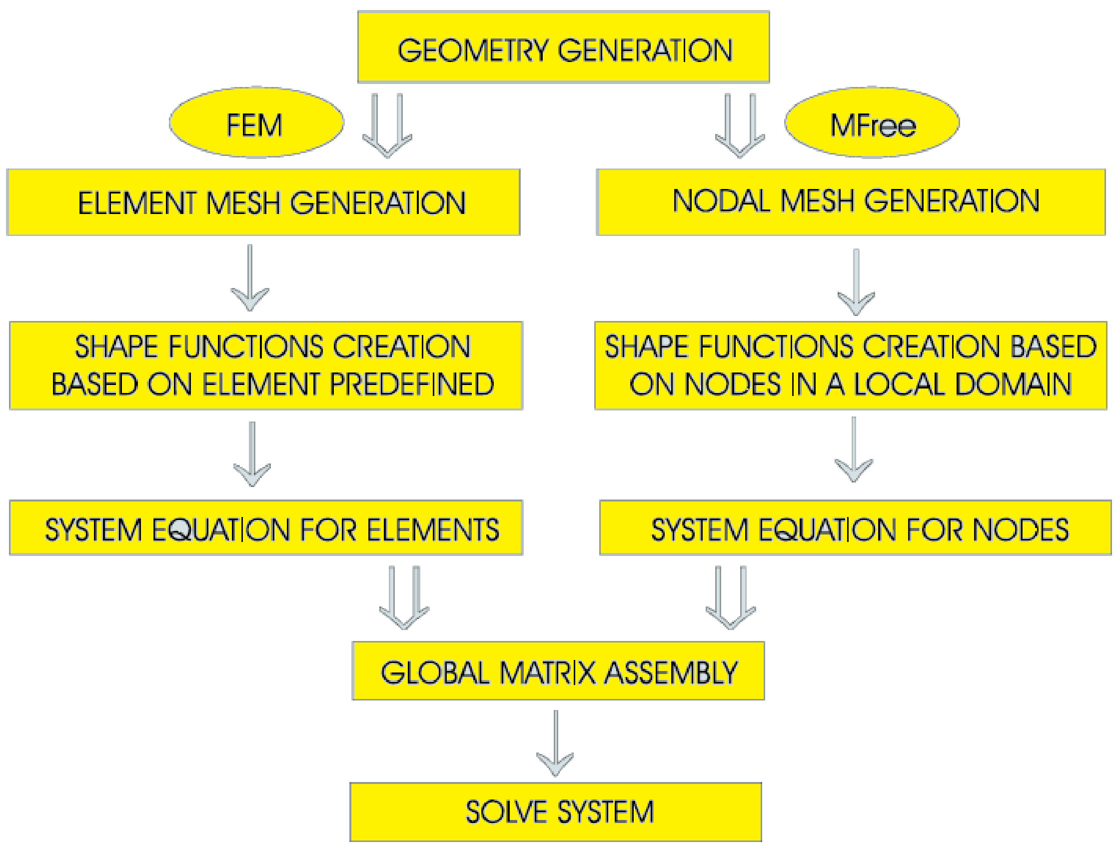



# Mesh Free Solving Equations

Numerical method depends on the kind of equations! Choices:

- Direct solver vs. iterative solver;
- Implicit method vs. explicit solver;
- Required accuracy;
- Speed of method;
- Stability of method;
- ...

# Mesh Free Solving Flow Chart



# Mesh Free Nodal Generation

- The nodes must represent both problem domain and boundary
- The nodes can be chosen arbitrary within reason
- The node distribution can be uniform or not.

# Mesh Free Function Properties

- Partition of unity (compulsory condition)

$$\sum_{i=1}^{n} \phi_i(\boldsymbol{x}) = 1$$

- Linear field reproduction (preferable condition)

$$\sum_{i=1}^{n} \phi_i(\boldsymbol{x}) x_i = \boldsymbol{x}$$

- Kronecker delta function property (preferable condition)

$$\phi_i(\boldsymbol{x}_j) = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

# Mesh Free MLS (Moving Least Square)

$$u^h(\boldsymbol{x}) = \sum_{j=0}^{m} p_j(\boldsymbol{x}) a_j(\boldsymbol{x}) = \boldsymbol{p}^T(\boldsymbol{x})\boldsymbol{a}(\boldsymbol{x})$$

with

$$\boldsymbol{p}^T(x) = \{1, x, x^2, ..., x^m\}$$
$$\boldsymbol{p}^T(x, y) = \{1, x, y, xy, x^2, y^2, ..., x^m, y^m\}$$
$$\boldsymbol{p}^T(x, y, z) = \{1, x, y, z, xy, yz, xz, x^2, y^2, z^2, ..., x^m, y^m, z^m\}$$

Features:

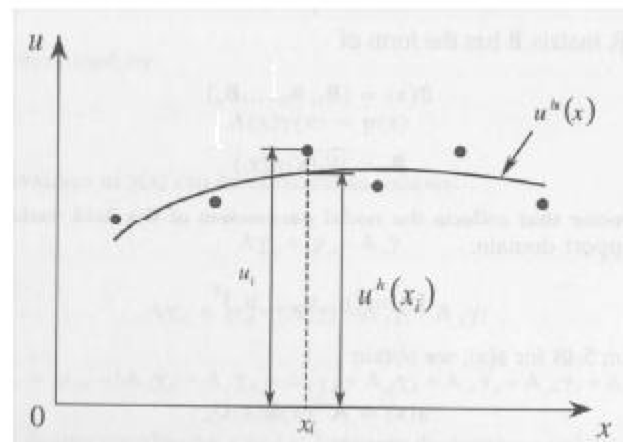
- finite series representation;
- ensures the approximated field function is continuous and smooth in the entire problem domain;
- capable of producing an approximation with the desired order of consistency.

# Mesh Free Residual Weight

$$u^h(\boldsymbol{x}, \boldsymbol{x}_i) = \boldsymbol{p}^T(\boldsymbol{x}_i)\boldsymbol{a}(\boldsymbol{x})$$

Weighted residual:

$$J = \sum_{i=1}^{n} W(\boldsymbol{x} - \boldsymbol{x}_i)[u^h(\boldsymbol{x}, \boldsymbol{x}_i) - u(\boldsymbol{x}_i)]^2$$
$$= \sum_{i=1}^{n} W(\boldsymbol{x} - \boldsymbol{x}_i)[\boldsymbol{p}^T(\boldsymbol{x}_i)\boldsymbol{a}(\boldsymbol{x}) - u_i]^2$$




In MLS approximation, at an arbitrary point $\boldsymbol{x}$, $\boldsymbol{a}(\boldsymbol{x})$ is chosen to minimize the weighted residual.

# Mesh Free Summary

- Mesh Free Methods are a respons to the limitations of Finite Element Methods.
- Mesh Free Methods do not use meshes, only nodes.
- The implementation of Mesh Free Methods differs from Finite Element Methods only in the shape function construction and node generation.
- The ideal Mesh Free Method is not found yet.

# Thank You!

Big Sucsses