אוניברסיטת בן-גוריון בנגב
Ben-Gurion University of the Negev

לשכת המהנדסים האדריכלים והאקדמאים במקצועות הטכנולוגיים בישראל

The 35th Israeli Conference on Mechanical Engineering – ICME 2018
Beer-Sheva, 9-10 October 2018

SNIC Bifurcation and its Application to MEMS

Shay Kricheli[1] and Oriel Shoshani[2]

[1]Ben-Gurion University of the Negev, Beer-Sheva, Israel, e-mail: krichelj@post.bgu.ac.il
[2]Ben-Gurion University of the Negev, Beer-Sheva, Israel, e-mail: oriels@bgu.ac.il

1. Abstract

This study focuses on a method to extract a frequency comb in mechanical means by an implementation of a beam that is exhibiting non-linear dynamics that is perturbed and analyzed for its transverse vibrations. The perturbation is an external harmonic driver with a chosen small amplitude and frequency, that when engaged with the unperturbed beam oscillations, causes it to reach SNIC bifurcation, rendering a frequency comb as desired. Computerized numerical simulations are run on it to check the results and compare them to the theory and desired outcome. The results agreed with the theory and produced the expected frequency comb.

Keywords: Nonlinear Dynamics, SNIC Bifurcation, Frequency Comb, Injection Pulling, Injection Locking.

2. Introduction

Injection pulling occurs when an interfering frequency source disturbs an oscillator but is unable to injection lock it, i.e., to synchronize it. The idea of harnessing that physical phenomenon in our advantage was covered in great detail and has numerous applications in physics, electrical and mechanical engineering [1] [2]. One interesting application of this interaction is to render a frequency comb - a series of discrete, frequency lines in the mathematical frequency domain centered around a carrier frequency. A frequency comb can be formally defined as:

$$f_n \equiv f_0 + n f_r \tag{1}$$

Where f_0 is the carrier frequency and f_r are the side bands, which are associated with the injection pulling phenomenon (Figure 1). Perhaps the most practical application of such a frequency comb is transition from one frequency range to another, e.g., the carrier frequency can be at the extremely high frequency (EHF) band, while the side band of the frequency comb can approach the radio frequency (RF) band.

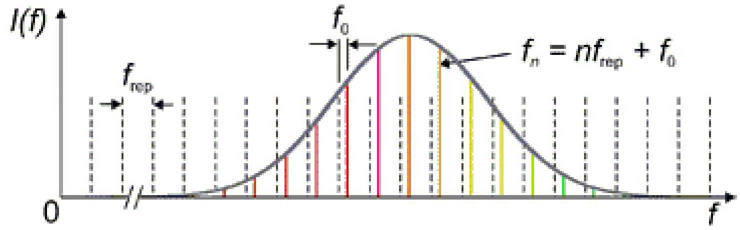

Figure 1 - Schematics of a frequency comb---the carrier frequency is at the center of the Lorentzian (orange) and there is a set of discrete frequency lines around it, i.e., a frequency comb

In this study we analyzed the manner in which this phenomenon occurs mechanically, when a mechanical oscillator is excited in such a way that a saddle-node on invariant circle (SNIC) bifurcation occurs and a frequency comb is produced. The implementation of this could relate to many fields, and in this study the relation of the subject to micro electro-mechanical systems (MEMS) was analyzed, hoping to provide some insight into the manner in which a fairly simple mechanical model can generate an output that other, more complex systems produced. The main setback in electrical systems is that they usually involve numerous components that each carry a degree of noise in the corresponding output signal they produce, which is often unwanted and usually disrupts the desired outcome and its applications. In that context, a mechanical model can be favorable in that it produces a reduced amount of noise and can be more precise in the results it will provide. The method of execution is the implementation of a closed-loop non-linear microbeam that is perturbed and analyzed for its transverse vibrations (Figure 2).

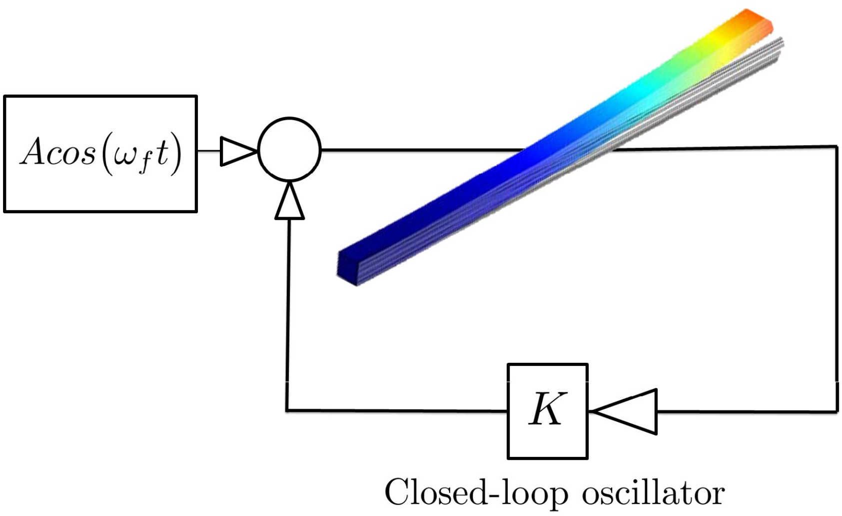

Figure 2 - Prototypical perturbed MEMS-based closed-loop oscillator - the mechanical beam serves as a frequency selective element and with an amplifer and phase shifter in a closed-loop configuration biulds the oscillator. In addition, there is an external perturbation that gaenerates the injection pulling effect

The perturbation is an external harmonic driver with a chosen small amplitude and frequency (which is slightly detuned from the beam eigenfrequency), that when engaged with the unperturbed beam oscillations, causes it, due to effects of injection pulling, to reach SNIC bifurcation, rendering a frequency comb as desired.

The beam dynamics in our case can be defined using the following PDE [3]:

$$\rho A \frac{\partial^2 u(x,t)}{\partial t^2} + 2\zeta \frac{\partial u(x,t)}{\partial t} - \tau[u(x,t)] \frac{\partial^2 u(x,t)}{\partial x^2} + EI \frac{\partial^4 u(x,t)}{\partial x^4} = \left[A\cos(\omega_d t) + \psi[u(x,t)]\right]\delta(x - x_0) \quad (2)$$

The beam is taken to be clamped-clamped so the boundary conditions are:

$$u(0,t) = u(l,t) = \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(l,t) = 0 \quad (3)$$

Using a single mode approximation and the Galerkin Projections method, we obtain the following non-dimensional ODE for the first mode dynamics:

$$\frac{d^2 T}{d\tau^2} + 2\tilde{\zeta}\frac{dT}{d\tau} + T + T^3 = \Phi \cos\left(\frac{\omega_d}{\omega_n}\tau\right) + \Gamma \frac{\frac{dT}{d\tau}}{\left|\frac{dT}{d\tau}\right|} \quad (4)$$

With the aid of the Averaging method, we showed that the problem can be further reduced into solving the Adler equation that has a known closed solution:

$$\dot{\phi} \approx B[K - \sin(\phi)] \quad (5)$$

3. Contributions

Theoretical analysis showed that the problem can be modelled using a non-linear equation of the beam, that translates to a form of the non-linear Duffing equation. While a solution to the Duffing equation is hard to obtain in practice due to mathematical difficulties, a slow evolution model is suggested that is composed of a pair of differential equations for the amplitude and the phase. Using several additional mathematical assumptions, the amplitude is seen to be related to the phase, while the phase equation solution is seen to be of the form of Adler's equation [4]. These assumptions ultimately reduce the entire behavior of the beam to a relatively simple solution of the Adler equation, which has a known analytical solution.

Computerized numerical simulations are run on it to check the results and compare them to the theory and desired outcome. The results agreed with the theory and produce the expected frequency comb, showing the assumptions to be valid in extracting the comb.

We solved Adler's equation to get the phase as a function of time and then calculated the solution's Fourier Transform, that transforms the function to the complex plane, in order to obtain the gain of the phase as a function of frequency.

Using the analytical solution to Adler's equation, one can experiment using the MATLAB numerical computing environment. Several simulations were made in order to witness the desired behavior graphically (Figure 3).

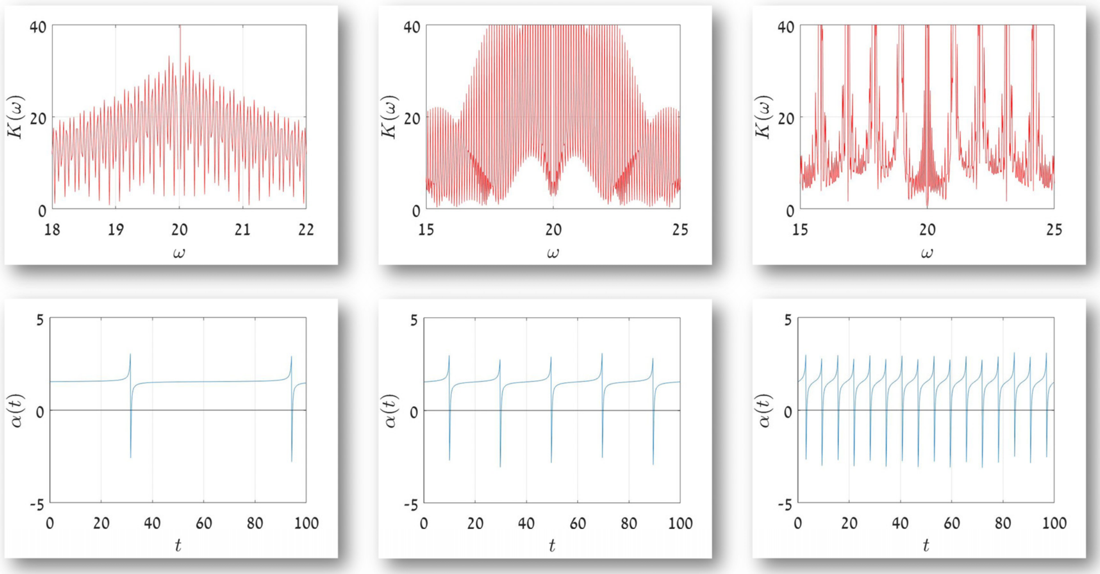

Figure 3 - Formation of the frequency comb frequency (upper panels) and time (lower panels) domains

4. Bibliography

[1] Oriel Shoshani, Daniel Heywood, Yushi Yang, Thomas W. Kenny, and Steven W. Shaw. "Phase Noise Reduction in a MEMS Oscillator Using a Nonlinearly Enhanced Synchronization Domain". Journal of Microelectromechanical Systems, 2016.

[2] Sudip Shekhar, Mozhgan Mansuri, Frank O'Mahony, Ganesh Balamurugan, James E. Jaussi, Joseph Kennedy, David J. Allstot, Randy Mooney, and Bryan Casper. "Strong Injection Locking in Low - LC Oscillators: Modeling and Application in Forwarded-Clock I/O Receiver". IEEE Transactions On Circuits And Systems, 2016.

[3] Ron Lifshitz and M. C. Cross. "Nonlinear Dynamics of Nanomechanical and Micromechanical Resonators". University of Novi Sad, Faculty of Technical Sciences, Serbia and University of Southampton, Institute of Sound and Vibration Research, Tel Aviv University, 2008.

[4] Robert Adler. "A Study of Locking Phenomena in Oscillators". Proceedings of the IRE, 1946.