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Fluid-solid interaction of minimally invasive procedures

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1. Abstract

We study the dynamics of a liquid-filled elastic tube as a response to the motion of a rigid cylinder inside it. The problem is governed by interaction between elastic deformation, the external force, fluidic pressure and the viscous flow at the annular gap between the cylinder and the tube. The insertion of the cylinder differs from its extraction out of the tube, yielding several physical regimes, such as viscous peeling, contact dynamics and system lockage. The mathematical model developed here could describe the mechanical behavior of minimally invasive medical procedures in biological vessels, such as endoscopy and catheterization.

Keywords: Fluid-solid interaction, Low-Reynolds number, Viscous-elastic dynamics, Cardio-vascular flows, Biological flows.

2. Introduction

In this paper, we analyze the dynamic response of a liquid-filled tube due to the forced motion of a coaxial rigid cylinder. This geometry is relevant to various minimally invasive medical procedures, in which solid devices are inserted into fluid-filled biological vessels. For example, new technologies for percutaneous revascularization involve inserting of cylindrical devices into a blocked blood vessel (Rogers & Laird 2007; Davis 2015); in laser angioplasty, a laser emitting catheter is inserted into an obstructed coronary artery in order to reopen it (Serruys et al. 1993). Additional relevant procedures are endoscopies of liquid-filled tubular body organs, e.g. cystoscopy and urethroscopy (Chew et al. 1996). Additionally, we will mention the frequently-used procedure of urinary catheterization, requiring the insertion of a catheter into a patient's bladder (Nacey & Delahijnt 1993).

In all the problems mentioned above – as a result of an externally forced motion of a cylindrical device, a flow-field is created, which in turn leads to viscous shear stresses and elastic deformations of the vessel. This variation in turn changes the flow-field. This research focuses on this coupling of the fluidic and the elastic effects of such geometries, i.e. the viscous-elastic dynamics. The aim of the research is to analyze the dynamic response of such geometry, in order to shed light on the mechanical behavior of various medical procedures, as presented above.

3. Modeling of the problem

We study Newtonian, incompressible, creeping flow in the annular gap between a rigid cylinder and a concentric elastic tube, as shown in figure 1-A. The cylinder represents a medical device which is used in procedures such those which were presented in the introduction. The elastic tube represents the relevant biological vessel (artery, urethra, etc.), and the viscous fluid represents the bio-fluid which is contained in the vessel, such as blood or urine. We are interested in the evolution in time of the length of the inserted cylinder; the gauge pressure

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inside the tube; and the radial deformation in time, which are represented by the parameters $l_p(t)$, $p_t(t)$ and $d_r(\tilde{z})$, respectively. The dynamical solution for these three parameters is obtained using three governing equations: the integral mass conservation, the force balance on the cylinder and the pressure-deformation equation. The ratio of pre-wetting thickness to characteristic radial deformations is defined by $\lambda_h = h_0/d_r^*$ and its value distinguishes between large-deformation and small-deformation solutions. We also distinguish between the case of a positive force $f_e(t) > 0$ applied on the cylinder, creating an insertion of the cylinder into the tube (see figure 1-B), and a negative force $f_e(t) < 0$, creating an extraction of the cylinder out of the tube (see figure 1-C).

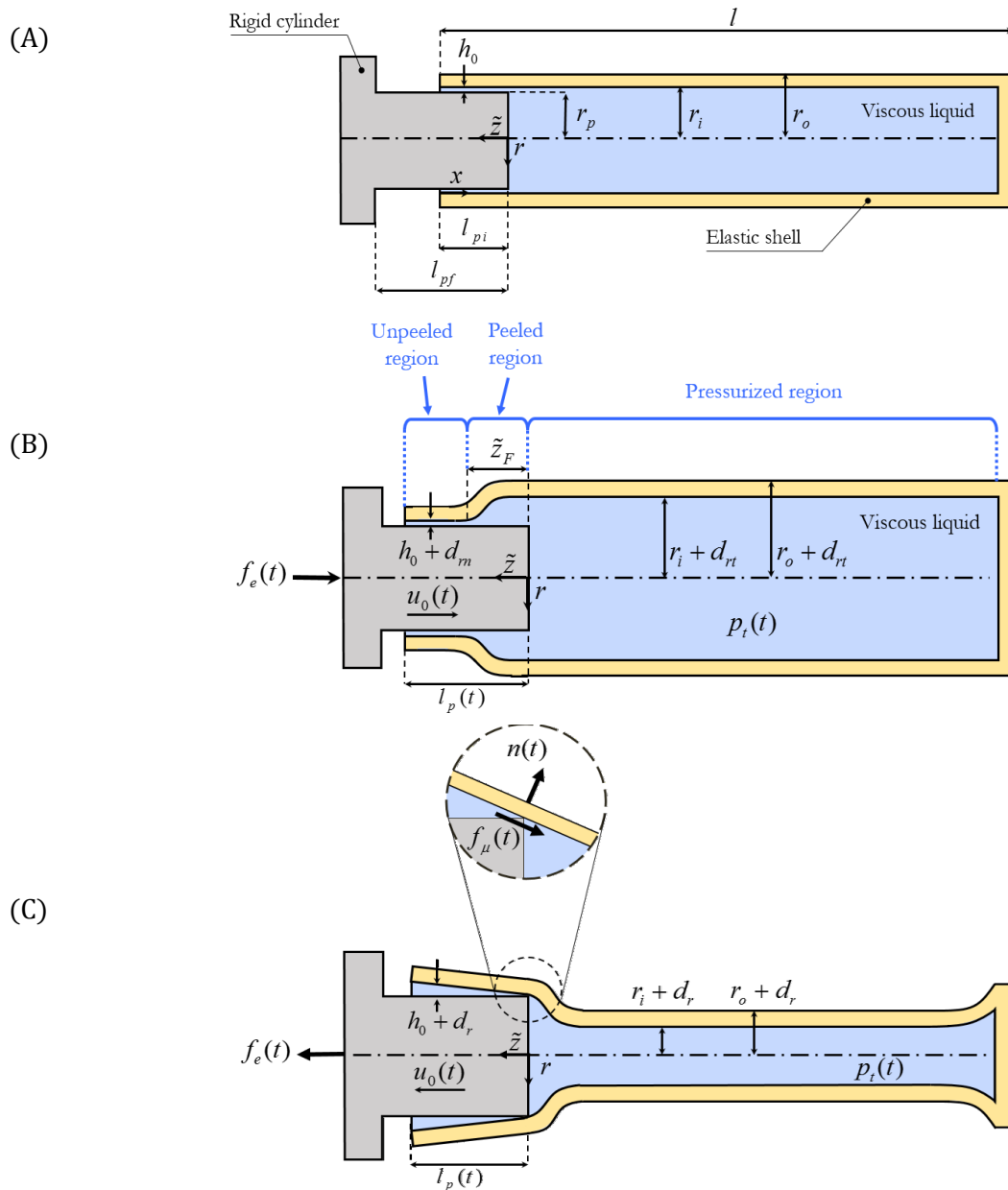


Figure 1: The analyzed geometry, consisting of an elastic tube and a concentric rigid cylinder with an annular gap between them. (A) The geometry in the unloaded state. (B) The deformed geometry for a positive force $f_e(t) > 0$, in the limit of large deformation. The geometry changes in time due to the advance of the cylinder, $l_p(t)$, and the propagation of the peeling front, $\tilde{z}_F(t)$. (C) The deformed geometry for a negative force $f_e(t) < 0$, for the limit of contact. As a result, a normal force $n(t)$ and a friction force $f_\mu(t)$ are created.

4. Dynamics solution

For the case of insertion, two limits were studied – small deformations and large deformations. At the limit of small deformation, the dynamics is dominated by the flow exiting through the annular gap between the tube and the cylinder. Using asymptotic methods, the analytical solution was obtained and it is demonstrated in figure 2.

However, at the limit of large elastic deformation as compared to gap's initial width, the response of the system is governed by viscous-peeling dynamics – a propagation of a viscous fluid beneath an elastic sheet. Such case involves an essential nonlinearity, and Van-Dyke's scheme and self-similarity solution were used to obtain the analytical solution for the dynamics of the system.

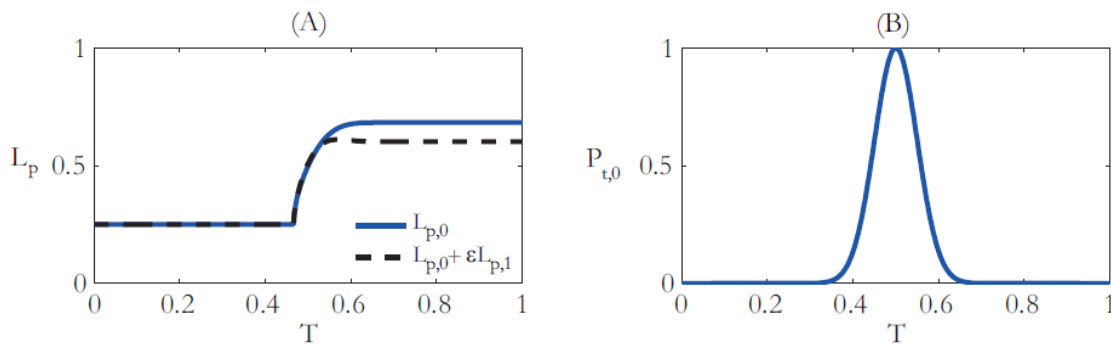


Figure 2: Dynamic response of the system for a Gaussian insertion force, in the limit of small deformations. (A) The penetrated length of the cylinder, L_p , is increasing monotonously over time, meaning that the cylinder moves inwards the tube. (B) The resulting gauge pressure, P_t , is of the form of the external force.

In the opposite case of an extraction of the cylinder, some more insights were obtained. While the dynamics of the system for small deformation is analogous to the insertion case and utilizes the same mathematical analysis, for large deformations the non-linearity of the extraction case yields an essentially different dynamics.

For a sufficiently large tension force, a new physical limit is created, in which the tube is in radial axisymmetric contact with the cylinder, so the gap is entirely closed and the entering of volume of fluid into the system is prevented. For this limit the advance of the cylinder depends also on the friction with the tube, leading to one of two possible modes: slip of the cylinder till its full exit from the tube, or a clamping state in which the cylinder is locked inside the tube.

Another limit to be analyzed is the near-contact dynamics. At this limit, the radial contact state is initially reached. Then, the external force is reduced so the deformation of the tube approaches the limit of the gap dimension. This creates a new fluid-structure interaction of the tube and the flow at the gap, which results in a moderate advance of the cylinder outwards the tube. This response is demonstrated in figure 3.

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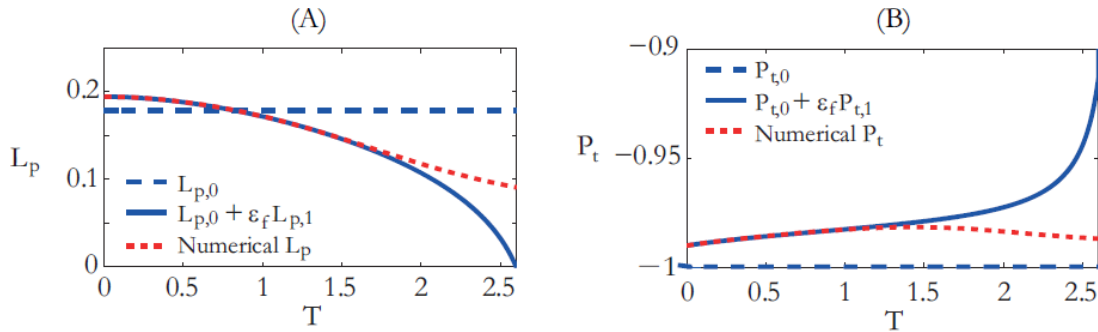


Figure 3: Dynamic response of the system at near-contact limit, for a tension force $F_e = 0.99 F_{e,contact}$ and initial state $l_{pi}/l > 2.5h_0/r_p$. The asymptotic expansions solution is compared with a numerical solution of the governing equations. (A) The penetrated length of the cylinder, L_p , is decreasing monotonously over time, meaning that the cylinder moves outwards the tube. (B) The resulting gauge pressure, P_t , starting at a negative value, is increasing.

5. Conclusions

This work examined the dynamic response of a liquid-filled tube due to the motion of a concentric rigid cylinder, intending to describe the mechanical behavior of minimally invasive medical procedures with a similar geometry. Using the governing equations of force balance, integral mass conservation and the pressure-deformation equation, an analytical solution was obtained for each limit and provided the penetrated length over time, as well as the pressure- and the deformation- fields.

For the insertion of the cylinder, the internal pressure is released through a narrow gap between two solid cylinders, so for sufficiently large forces the upper elastic tube is peeled. In the opposite case of cylinder's extraction out of the tube we found that the cylinder exits the tube in a finite time for sufficiently small or large forces, while for an intermediate range of forces the cylinder is locked inside the tube. This occurs due to the negative gauge pressure, which reduces the gap between the cylinder and the tube till a radial contact of the two solids. The analysis of the near-contact limit showed that the contact between the tube and the cylinder is not stable for reducing the force to a value smaller than the exact contact force. This is reflected by the motion of the cylinder outwards the tube, till complete exit or till zeroing of the internal pressure.

6. References

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