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Green's Function Method for Solving the Separated Two-Phase Flow in Inclined Tubes by Superposition

Ofer Eyal[1] and Ayelet Goldstein[2]

ORT Braude College, P.O.B. 78, Karmiel 21982, Israel

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1. Abstract

A new method for solving a separated, fully developed laminar flow in inclined tubes with a curved interface is suggested. Apart from the solution that is available in the literature Goldstein et al, 2015, 2017)[1], where for each configuration has a compatible solution with a conformal mapping to the bipolar or unipolar coordinate, this paper suggests a single solution that is automatically tailored for the whole range of the configurations of the separated flow with Cartesian or polar coordinate. The solution is built up by the superposition of the contributions of the Driving Force Density (DFD) to the flow field. Surface densities of driving forces are distributed on the cross section of the tube and serves mathematically as the inhomogeneous part of the Poisson equation. Moreover, a line density must be added on the interface in to obey a continuity of the shear stresses on the interface (that means discontinuity of the velocity gradients). This way of solution is intuitive with an insight sense.

2. Introduction

Separated flow is considered a basic flow pattern in horizontal inclined gas-liquid and liquid-liquid systems of a finite density difference, since for some range of sufficiently low flow rates, the two phases tend to segregate. The separated flow includes all the configurations such as stratified and core annular flow. Counter-current is encountered in the process industry, in various mass transfer and direct contact heat transfer systems, and is feasible only in inclined systems [2]. In principle, exact analytical solutions for the two-dimensional velocity profiles and shear stress profiles in pipe flow can be obtained only for fully developed laminar flows. Such solutions are of practical significance mainly for studying two-phase flow in small diameter pipes or liquid-liquid flows. The range of operational conditions where stratified flow can be established in min micro channels was found to be affected by various factors, such as the inlet device (and premixing), tube inclination, surface wetting conditions due to tube surface material and start-up procedure (e.g., Dreyfus et al., 2003[3]; Matsumoto et al., 2007[4]; Salim et al., 2008[5]; Wang et al., 2012; Ami et al., 2012[6]; Mehta and Banerjee[7], 2014; Goldstein et al., 2015[8]; Maklakov et al., 2017[9]). A complete set of solutions for laminar stratified flow in inclined pipes, which include all possible configurations of stratified flow with concave and convex smooth interfaces, have been recently presented by Goldstein et al;2015[8], using bipolar and unipolar coordinate systems. That set of solutions includes also the case of fully eccentric core-annular _ow, which represents an extreme case of stratified flow with either concave or convex interface. Packham and shail, 1971 found an analytical solution only for special cases such as the diametric interface that splits the pipe into half. This solution have been extended by Maklakov et al., 2017. They solved analytically for stratified flow but limited for wetting angle of α= π/2 result in the same surface tension between the wall and for each phase to be equal. The mathematical advantage of wetting angle π/2 is that one part of the disk can be transform by inversion with respect to the interface circle into the other part of the disk, and the harmonic part of the solution in one phase can serve also as solution in the other phase. The combination of inversion followed by the logarithmic function as used at Maklakov et al., 2017 has been actually expressed as the passage to the bipolar coordinate that was used at Goldstein et al., 2015, Biberg[10] and Halvorsen., 2000 and Rovinsky[11] et al., 1996. Goldstein et al., 2017 obtained the analytical solution for the CAF configuration with the bipolar coordinate system.

All the above solutions were mathematically obtained via the Navier-Stokes equations with different methods. In the present study the solution is decomposed intuitively from contributions to the flow from its originators, with more simple and real picture, as the electrostatic analogue of producing electric potential by electric charges Flow.

Dividing the disk into two parts: the convex part (core) that will be indexed by "c" and the concave part that will be indexed by "a" (annulus). Referring to such STF's geometry that is shown in Fig.1, two polar coordinates were chosen for simplifying the solution instead of dealing with conformal mapping as were presented in other studies.

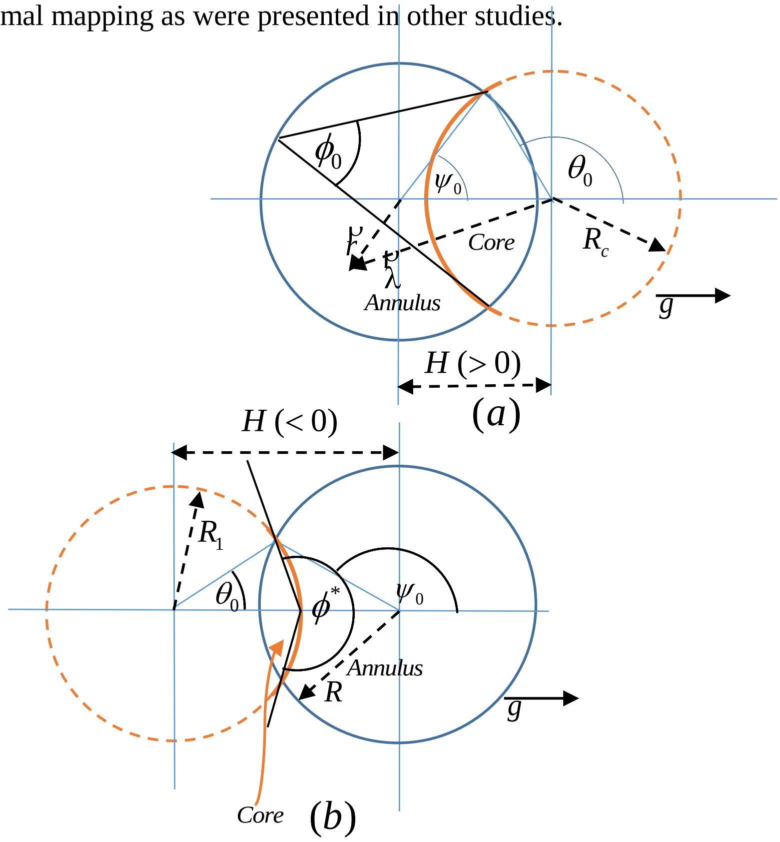


Figure 1: STF configuration Flow: The tube with radius R is divided to two cross section areas *c* and *a* by the interface which is apart of another circle with radius *Rc*, where $2\pi-\psi_0 < \psi < \psi_0$. Two polar coordinates ($r;\ \psi$) and ($\ell;\theta$). The gravity acceleration is oriented to the right side;(a) the core's center is located at the right hand side, so the $H > 0$ (a convex interface); (b)the core's center is located at the left hand side, so the $H < 0$ (a convex interface).

N-S Equation for each phase:

$$\nabla^2 \tilde{u} = \left\{ \begin{array}{lcl} \frac{8\tilde{P}_a}{\tilde{\mu}} \equiv \alpha_a \tilde{P}_a & : & \vec{r} \in a \\ 8\tilde{P}_c \equiv \alpha_c \tilde{P}_c & : & \vec{r} \in c \end{array} \right\}$$

3. Two Phase Flow Configurations: STF and CAF's

Since the axial flow is fully developed, the *z* component of the velocity can be considered as a scalar in an effective two dimensional flow. In this paper, the velocity distribution is created by the driving force and can be considered as an analogous electrical potential that is created by charge densities, correspondingly. By the superposition law, this *z* component of the velocity field is decomposed of fields created by the following density contributions:

$$\tilde{u} = \underbrace{\frac{\alpha_a \tilde{P}_a}{4}\left(r^2 - R^2\right) + \frac{1}{4}\left(\alpha_c \tilde{P}_c - \alpha_a \tilde{P}_a\right)\left(\Theta(c)\left(\ell^2 - R_c^2\right) + R_c^2 \Theta(a) \ln \frac{\ell^2}{R_c^2}\right) - \tilde{u}_{comp} +}_{u_{contin}}$$
$$+ \int_{\theta_{min}}^{\theta_{max}} R_c d\theta' G(\vec{r}, \vec{r}') f(\theta')$$

4. Validation to Literature

Validation is presented to the solution that is available in the literature for inclined flow in the all separated flow configuration. In Fig.2 is shown the comparison between the two solutions for counter-current con-centric CAF flow (Ullmann and Brauner.2003) for a light core, where 2a presents the velocity distribution at the center line and (b) presents the solution at this paper at the all tube cross section. Fig.2 presents an eccentric counter-current CAF flow where the core is light directs to upward flow. Fig.3 presents a stratified flow for the $\phi * = 1.5\pi$. Fig.4 demonstrates the counter-current flow for a heavy core flowing downward, where Fig.4a is the solution that is available in the literature obtained by the bipolar coordinate system (ayelet et al.,2015) and Fig.4b is the suggested solution for the collection of charges contribution in the polar and Cartesian coordinate system.

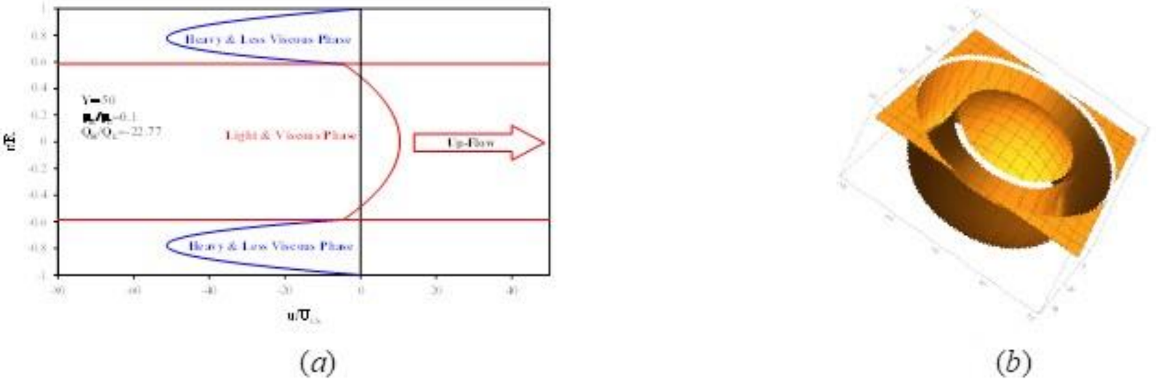


Figure 1: *Velocity Profile Concentric CAF core is the light phase:* $\mu = 0.1; q = -22.77; Y = -50; \epsilon = 0.6519$ *(a)at the center-line; (b)the entire surface's tube*

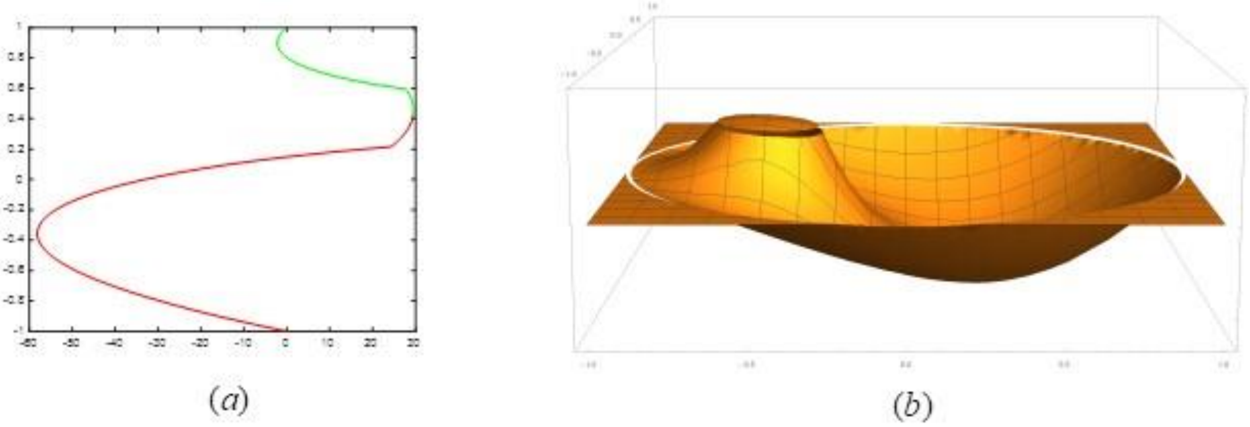
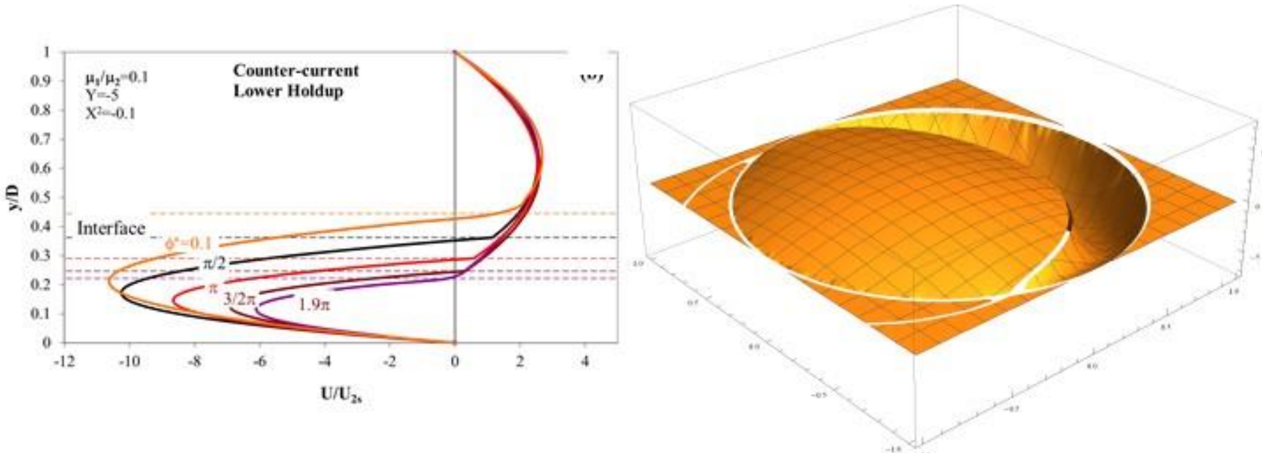


Figure 4.2: *Velocity Profile Eccentric CAF-core is the light phase:* $\mu = 0.1; q = -22.77; Y = -50; \epsilon = 0.9639; E = 0.5$ *(a)at the center-line; (b)the entire surface's tuve*



(a)

(b)

Figure 3: *Velocity Profile Stratified flow:* $\mu = 0.1$;$q = -22.77$;$Y = -50$;$\epsilon = 0.6519$ *(a)at the center-line;*

(b)the entire surface's tuve

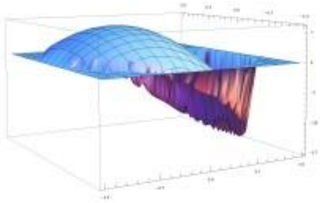


Figure 4: *Velocity Profile of full eccentric CAF*

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